

Written Assignment 4

2.5 points each

5.1

Algebraic

For the following exercises, rewrite the quadratic functions in standard form and give the vertex.

1. $y = 4x^2 + 5x - 7$

For the following exercises, determine whether there is a minimum or maximum value to each quadratic function. Find the value and the axis of symmetry.

2. $y = \frac{1}{3}x^2 - 4x + 9$

For the following exercises, rewrite the quadratic functions in standard form and give the vertex. Determine whether there is a minimum or maximum value to each quadratic function. Find the value and the axis of symmetry. Determine the domain and range of the quadratic function.

3. $f(x) = \frac{-1}{2}x^2 + 4x - 2$

For the following exercises, use the vertex (h, k) and a point on the graph (x, y) to find the general form of the equation of the quadratic function.

4. $(h, k) = (-1, 3), (x, y) = (-2, 6)$

Real-World Applications

5. An athletic stadium holds 105,000 fans. With a ticket price of \$22, the average attendance has been 32,000. When the price dropped to \$16, the average attendance rose to 50,000. Assuming that attendance is linearly related to ticket price, what ticket price would maximize revenue? Round ticket price to the nearest ten cents.

5.2

Algebraic

For the following exercises, identify the function as a power function, a polynomial function, or neither.

6. $f(x) = 4(x^3)^3$

For the following exercises, find the degree and leading coefficient for the given function if it is a polynomial. If it is not a polynomial then state so.

7. $f(x) = (2x^2 - 5)^2 + (x - 3)^2 + 5$

For the following exercises, find the intercepts of the functions.

8. $g(x) = (3x^2 - 10x - 8)(x + 3)$

Technology

For the following exercises, graph the polynomial functions using a calculator or a graphing utility. Based on the graph, determine the intercepts and the end behavior.

9. $f(x) = x^6 - 36$

Real-World Applications

For the following exercises, use the written statements to construct a polynomial function that represents the required information.

10. A cube has an edge of 2.25 feet. The edge is increasing at the rate of 1.25 feet per hour. Express the volume of the cube as a function of h , the number of hours elapsed.

5.3

Algebraic

For the following exercises, find the x-intercepts of the polynomial functions. Express all intercepts in point (ordered pair) form.

11. $f(x) = 5x^3 - 20x^2 + 2x - 8$

For the following exercises, use the Intermediate Value Theorem to confirm that the given polynomial has at least one zero within the given interval

12. $g(x) = -3x^3 - 12x^2 + 11x$ between $x = -5$ and $x = -4$

Graphical

For the following exercises, graph the polynomial functions. Note x- and y-intercepts, multiplicity, and end behavior.

13. $f(x) = x(x + 4)(x - 4)(x + 7)$

For the following exercises, use the given information about the polynomial graph to write the equation.

14. Degree 3; zeros at $x = -1, x = 4, x = -5$; passes through the point $(2, -84)$

Real-World Applications

15. A cylinder has a radius of $2x + 3$ units and a height of 5 units greater than the radius. Express the volume of the cylinder as a polynomial function.

5.4

For the following exercises, use long division to divide. Specify the quotient and the remainder.

16. $(x^2 - 6x + 4) \div (x - 2)$

For the following exercises, use synthetic division to find the quotient.

17. $(18x^3 + 3x^2 - 33x + 20) \div (3x + 5)$

For the following exercises, use synthetic division to determine whether the first expression is a factor of the second. If it is, indicate the factorization.

18. $x - 3$; $2x^2 + 7x - 12$

For the following exercises, use synthetic division to find the quotient and remainder.

19. $\frac{2x^3 - 6x^2 + 5}{2x + 3}$

Extensions

For the following exercises, use synthetic division to determine the quotient involving a complex number.

20. $\frac{x^2 + 3}{x - i}$

5.5

Algebraic

For the following exercises, use the Remainder Theorem to find the remainder which is the value of $f(x)$ at $x = k$. Then evaluate $f(x)$ at $x = k$ to verify remainder and $f(x)$ at $x = k$

are the same. Show all work for both synthetic division and the evaluation of $f(x)$ at $x = k$.

21. $(2x^4 - 5x^3 + 2x^2 - x - 1) \div (2x + 1)$

For the following exercises, use the Factor Theorem to find all real zeros for the given polynomial function and one factor.

22. $f(x) = 6x^3 - 13x^2 - 73x - 60$; $2x + 3$

For the following exercises, use the Rational Zero Theorem to find all real zeros. State all the possible rational zeros.

23. $4x^3 - 7x - 3 = 0$

For the following exercises, find all complex solutions (real and non-real).

24. $4x^3 - 8x^2 + 9x - 18 = 0$

Graphical

Use Descartes' Rule to determine the possible number of positive and negative solutions. Make a table of possible real/complex solutions as shown in example 8 of Section 5.5. Then graph to confirm which of those possibilities is the actual combination.

25. $f(x) = x^4 - 2x^3 - 3x^2 + 5x - 3$

Technology

For the following exercises, use your graphing calculator or a graphing utility program to graph the polynomial function. Based on the graph, find the rational zeros. All real solutions are rational.

$$26. f(x) = 12x^4 - 11x^3 + 33x^2 - 44x - 60$$

Extensions

For the following exercises, construct a polynomial function of least degree possible using the given information.

$$27. \text{Real roots: } -2, 2, -3 \text{ and } f(3) = 10$$

Real-World Applications

For the following exercises, find the dimensions of the right circular cylinder described.

$$28. \text{The radius is half height. The volume is } 128\pi \text{ cubic feet.}$$

5.6

Algebraic

For the following exercises, find the domain of the rational functions.

$$29. f(x) = \frac{2x^2+5x-12}{4x^2+13x-35}$$

For the following exercises, find the x- and y-intercepts for the functions.

$$30. f(x) = \frac{3x^2-4x-8}{8x^3+64}$$

For the following exercises, find the slant asymptote of the functions.

$$31. f(x) = \frac{3x^3+4x^2-7x+12}{x^2+3}$$

For the following exercises, find the horizontal intercepts, the vertical intercept, the vertical asymptotes, and the horizontal or slant asymptote of the functions. Use that information to sketch a graph.

32. $f(x) = \frac{3x^2 - 7x - 20}{x^3 - 2x^2 - 29x - 42}$

For the following exercises, write an equation for a rational function with the given characteristics.

33. Vertical asymptotes at $x = -7$ and $x = \frac{3}{2}$, x-intercepts at $(-6, 0)$ and $(3, 0)$,
y-intercept at $(0, \frac{6}{7})$, horizontal asymptote at $y = \frac{1}{2}$

5.7

Algebraic

For the following exercises, find the inverse of the function on the given domain.

34. $f(x) = (x + 6)^2$, $[-6, \infty)$

For the following exercises, find the inverse of the functions.

35. $f(x) = 4x^3 + 5$

For the following exercises, find the inverse of the functions.

36. $f(x) = \sqrt{3x + 4}$

Graphical

For the following exercises, find the inverse of the function and graph both the function and its inverse.

37. $f(x) = 3x^2 + 5, x \geq 0$

5.8

Algebraic

For the following exercises, write an equation describing the relationship of the given variables.

38. y varies directly with the cube of x and inversely with the square of z , when
 $x = 4$ and $z = 2, y = 8$

Numeric

For the following exercises, write an equation describing the relationship of the given variables. Then use the given information to find the unknown value.

39. y varies jointly with the fourth root of a and the square of b and inversely with the cube of c , when $a = 81$ and $b = 7$ and $c = 3, y = 49$.

Real-World Applications

For the following exercises, use the given information to answer the questions.

40. A person's body mass index (BMI) b varies directly to a person's weight in pounds and inversely with the square person's height h in inches. If a 6 foot tall person's BMI is 32 and their weight is 236 pounds, write an equation describing the relationship of the given variables. NOTE: Keep k as an exact fractional value not a decimal approximation.